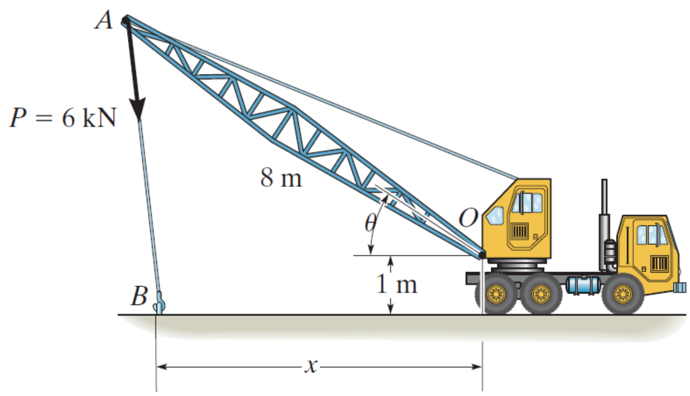
Problem 4-11

The towline exerts a force of $P = 6$ kN at the end of the 8-m-long crane boom. If $\theta = 30°$, determine the placement x of the hook at B so that this force creates a maximum moment about point O. What is this moment?



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Solution

Treat O as the origin of an xyz-coordinate system and write the position vectors to points A and B.

$$\mathbf{r}_A = 8\langle -\cos 30°, \sin 30°, 0\rangle \text{ m}$$

$$\mathbf{r}_B = \langle -x, -1, 0\rangle \text{ m}$$

Use these to write a formula for the force acting from A to B.

$$\mathbf{P} = P\hat{\mathbf{u}}_{\text{AB}} = 6000\frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} \text{ N} = 6000\frac{\langle -x + 8\cos 30°, -1 - 8\sin 30°, 0\rangle}{\sqrt{(-x + 8\cos 30°)^2 + (-1 - 8\sin 30°)^2 + 0^2}} \text{ N}$$

$$= 6000\frac{\langle -x + 4\sqrt{3}, -5, 0\rangle}{\sqrt{(-x + 4\sqrt{3})^2 + 25}} \text{ N}$$

Now calculuate the moment of this force about O.

$$\mathbf{M}_P = \mathbf{r}_A \times \mathbf{P} = 8\langle -\cos 30°, \sin 30°, 0\rangle \times 6000\frac{\langle -x + 4\sqrt{3}, -5, 0\rangle}{\sqrt{(-x + 4\sqrt{3})^2 + 25}} \text{ N} \cdot \text{m}$$

$$= \frac{48\,000}{\sqrt{(-x + 4\sqrt{3})^2 + 25}}\langle -\cos 30°, \sin 30°, 0\rangle \times \langle -x + 4\sqrt{3}, -5, 0\rangle \text{ N} \cdot \text{m}$$

$$= \frac{48\,000}{\sqrt{(-x + 4\sqrt{3})^2 + 25}}\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ -\cos 30° & \sin 30° & 0 \\ -x + 4\sqrt{3} & -5 & 0 \end{vmatrix}$$

Evaluate the determinant.

$$\mathbf{M}_P = \frac{48\,000}{\sqrt{(-x+4\sqrt{3})^2+25}}\left[\frac{1}{2}(x+\sqrt{3})\ \mathrm{N}\cdot\mathrm{m}\right]\hat{\mathbf{z}}$$

$$= (24\,000\ \mathrm{N}\cdot\mathrm{m})\frac{x+\sqrt{3}}{\sqrt{(-x+4\sqrt{3})^2+25}}\,\hat{\mathbf{z}}$$

In order to find the value of x that maximizes the moment, take the derivative of this function of x and set it equal to zero.

$$\frac{d}{dx}\left[\frac{x+\sqrt{3}}{\sqrt{(-x+4\sqrt{3})^2+25}}\right] = 0$$

$$\frac{\left[\frac{d}{dx}(x+\sqrt{3})\right]\sqrt{(-x+4\sqrt{3})^2+25} - \left[\frac{d}{dx}\sqrt{(-x+4\sqrt{3})^2+25}\right](x+\sqrt{3})}{(-x+4\sqrt{3})^2+25} = 0$$

$$\frac{(1)\sqrt{(-x+4\sqrt{3})^2+25} - \left\{\frac{1}{2}\left[(-x+4\sqrt{3})^2+25\right]^{-1/2}\cdot\frac{d}{dx}[(-x+4\sqrt{3})^2+25]\right\}(x+\sqrt{3})}{(-x+4\sqrt{3})^2+25} = 0$$

$$\frac{\sqrt{(-x+4\sqrt{3})^2+25} - \left\{\frac{1}{2}\left[(-x+4\sqrt{3})^2+25\right]^{-1/2}\cdot 2(-x+4\sqrt{3})\cdot\frac{d}{dx}(-x+4\sqrt{3})\right\}(x+\sqrt{3})}{(-x+4\sqrt{3})^2+25} = 0$$

$$\frac{\sqrt{(-x+4\sqrt{3})^2+25} - \left\{\left[(-x+4\sqrt{3})^2+25\right]^{-1/2}\cdot(-x+4\sqrt{3})\cdot(-1)\right\}(x+\sqrt{3})}{(-x+4\sqrt{3})^2+25} = 0$$

$$\frac{[(-x+4\sqrt{3})^2+25]+(-x+4\sqrt{3})(x+\sqrt{3})}{[(-x+4\sqrt{3})^2+25]^{3/2}} = 0$$

Multiply both sides by $[(-x+4\sqrt{3})^2+25]^{3/2}$ and solve for x.

$$[(-x+4\sqrt{3})^2+25]+(-x+4\sqrt{3})(x+\sqrt{3}) = 0$$

$$85 - 5\sqrt{3}x = 0$$

$$x = \frac{17}{\sqrt{3}}\ \mathrm{m} \approx 9.81\ \mathrm{m}$$

Plugging this value of x into the formula for $\mathbf{M}_P$ yields

$$\mathbf{M}_P = 4.80\times 10^4\ \mathrm{N}\cdot\mathrm{m}$$

for the maximum moment of force $\mathbf{P}$ about O.